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FINITE ELEMENT MODELLING OF STRATIFIED FLOW IN ESTUARIES AND RESERVOIRS

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SUMMARY

Experiences relating to the application of finite element models for laterally averaged stratified flow are discussed and modifications to the basic approach are suggested that alleviate these difficulties. An example problem is used to demonstrate the revised model and to make a preliminary assessment of the hydrostatic pressure assumption when applied to reservoir analysis.

KEY WORDS Finite Element Reservoir Hydrostatic Pressure Numerical Accuracy Stratified Flow

INTRODUCTION

There are now a growing number of models that are applicable to the simulation of stratified flow in reservoirs and estuaries. These models use both finite difference and finite element algorithms and simulate time transients using both explicit and implicit schemes.[1–4]

Frequently the descriptions of applications of models of this type show details of successful simulations but do not detail all the problems that arose. It is the purpose of this paper (a) to review some of the difficulties experienced using the previously described RMA-7 model,[1] (b) to present some alternatives for resolution of these problems both for the solution technique and for the basic structure of the equations and (c) to show their impact when applied to a demonstration reservoir.

The principal difficulties that will be reviewed include those associated with the free surface, which creates an uncertainty as to the geometric extent of the problem, the specification of satisfactory boundary conditions where flow enters and leaves the system, and the numerical sensitivity of pressure and density variations that, coupled with the aspect ratio of typical problems, can lead to unstable results.

Solutions that are evaluated include geometric transformations that fix the geometric extent of the problem, reformulation of the pressure term and incorporation of the hydrostatic pressure assumption.

In order to define the problem, the basic equations used in the RMA-7 model will be presented, together with a description of the finite element implementation.

GOVERNING EQUATIONS AND SOLUTION METHOD

The Navier–Stokes equations for two dimensional flow in association with the convection–diffusion equation for heat form the basic equations that describe the stratified flow system (in estuarial cases the heat flow equation may be augmented or replaced by a salinity transport

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equation of the same form). They may be written as:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]+\frac{\partial p}{\partial x}-\varepsilon_{xx}\frac{\partial^2 u}{\partial x^2}-\varepsilon_{xy}\frac{\partial^2 u}{\partial y^2}=0,$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]+\frac{\partial p}{\partial y}+\rho g-\varepsilon_{yx}\frac{\partial^2 v}{\partial x^2}-\varepsilon_{yy}\frac{\partial^2 v}{\partial y^2}=0,$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$C_\rho\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]-D_{xx}\frac{\partial^2 T}{\partial x^2}-D_{yy}\frac{\partial^2 T}{\partial y^2}=0,$$

Subject to an equation of state that relates temperature and density $F(\rho, T)=0$, where u, v are the velocity components in the Cartesian directions, x horizontally and y vertically upwards. g is acceleration due to gravity, positive downwards, p is the pressure, ρ is the density, T is the temperature, C is the specific heat and ε_{xx}, ε_{yy}, ε_{yx}, ε_{yy}, D_{xx} and D_{yy} represent the turbulent eddy and diffusion coefficients, respectively.

A weighted residual scheme is used to develop the finite element algorithm for the governing equations. The formulation has been described in detail previously[1] and for present purposes only a brief overview is given here.

The basic elements used are isoparametric curved triangles and quadrilaterals with 6 and 8 nodes, respectively. The model uses mixed interpolation with quadratic shape functions for velocity components and temperature and linear functions for pressure.

The model uses a Newton scheme to incorporate the non-linear terms with several iterations per time step and an implicit Crank–Nicolson weighting scheme that steps forward through time. However a single solution step, perhaps with several iterations, can be used to simulate a system that is in steady state. Examples of earlier solutions using this model have been presented in Reference 2.

PREVIOUS EXPERIENCE

The earliest test cases and applications of this model focused on experimental data available for laboratory size experiments, thus the space scale of the application was relatively limited and for stratified flow cases the velocities were very small. Later applications were made to full size prototype systems of lakes and estuaries. In these applications the final results that were obtained showed good correlation with available data, but problems appeared in several areas. The sections that follow discuss these difficulties and the steps undertaken to resolve them.

TREATMENT OF THE FREE SURFACE

In the general time-dependent case the free surface moves with time. For reservoirs the surface slopes are usually very small and the elevation of the water surface is approximately that at the dam. For estuaries, however, the spatial variation of surface elevation is much larger when, for example, tidal waves are traversing the system. The geometric extent of the system itself varies during the simulation and becomes a continuous parameter of the problem. Two schemes were tested to compute the free surface. Both are limited to relatively small changes in the free surface during a time step, and require a finite element mesh that varies in time. In each method an initial estimate of the free surface is made, and successive corrections used to estimate the true level.

(a) In the first approach a correction step consists of computing a solution where the pressure at the surface is set equal to zero, but where the velocity components are unconstrained. The free surface co-ordinates are then adjusted so that the normal velocity of the surface matches the rate of change of surface location, simultaneously accounting for any storage change during the time step. That is, the continuity equation is solved for the surface profile. In steady state cases, the surface is adjusted by assuming that the velocity normal to the water surface acts under gravity alone and modifying the elevation to the level associated with zero vertical velocity.

(b) The second approach develops a correction step by computing a solution for the normal velocity at the free surface so that it is equal to the rate of change of the free surface (computed from the current and previous elevations). That is, the velocities are specified but the magnitude of the pressure is a dependent variable. For steady state cases the normal velocity is specified as zero. The free surface is then adjusted so that the pressure at the surface is reduced to zero.

Both these schemes appear attractive, but in prototype systems, even for steady state problems, the first scheme repeatedly became unstable and the second approach showed very slow convergence characteristics. In actual applications both methods were too expensive to use.

Subsequent prototype applications of the model have used the latter scheme, but resorted to outside estimation in order to define the free surface and discontinued any geometric variation within the time step. This method is thus comparable to the 'rigid lid method' used in many three dimensional methods and incorporates some inconsistency in the overall satisfaction of continuity for the system.

An alternative procedure can be constructed that reduces these difficulties. This method transforms the basic equations to create a constant geometric configuration and makes the free surface elevation a dependent variable of these modified equations. The difficulty with this method is that transformations of this type complicate the basic equations, the computer programming and the convergence of the solution. This approach has been implemented by the author[5] for solution to the three-dimensional flow equations written in shallow water form. In this case there is only one pressure or head variable at a particular horizontal location.

SPECIFICATION OF ENTRY/EXIT BOUNDARY CONDITIONS IN ESTUARIAL APPLICATIONS

Specification of entry/exit boundary conditions for a fully stratified problem requires knowledge either of the velocity distribution with depth at entry and/or exit or knowledge of the pressure distribution. In practice such data are seldom available for both boundaries. Typically, available data consist of upstream flow and temperature distribution, and surface elevations downstream, sometimes with a temperature distribution. The modeller must then generate some kind of expansion of the known conditions to adequately represent the system. In estuarial applications on an incoming tide, the temperature conditions must be estimated together with a pressure distribution (using for example, a hydrostatic assumption), but on an outgoing tide the temperature will be determined by the convective transport and because of the unknown density it is not possible to estimate beforehand the pressure distribution (even under the hydrostatic assumption). This difficulty can be reduced by using a separate depth-averaged simulation to estimate the overall flow rates at the boundaries and then defining a vertical distribution based on engineering experience. The resulting velocities then provide the modeller with an approximation to the boundary conditions.

It is noteworthy that the depth-averaged simulation can also be used to specify the approximate free surface needed to resolve the free surface difficulty discussed in the section above, even in estuarial cases.

An alternative and, in a sense, more satisfactory solution is to extend the system to be analysed so that it includes boundary conditions that are more uniform. For example, beyond the extent of stratification. Such an approach may not be feasible at all times, however, either because of the specifics of the problem or the economics of computer modelling.

NUMERICAL ACCURACY WHEN OPERATING WITH THE PRESSURE TERM

The vertical momentum equation is a balance between the inertia and turbulence terms, the differential gradient associated with pressure and the self weight terms. These latter two terms approximately cancel each other out and computations of pressure gradients in the vertical and also in fact the horizontal directions are strongly influenced by small differences of large numbers. This problem is exacerbated by the numerical problem introduced by the high aspect ratio of the finite element network typically necessary for these problems.

A second and more significant problem arises because the model uses mixed interpolation with linear pressures and quadratic velocity and temperature distributions. When an irregular network in the vertical direction is defined (see Figure 1 where the elevations of C and D are different and where triangular elements are used) it is possible to generate circulation for a system that is structured with exactly horizontal layers of linearly varying density that should be exactly at rest.

For example, consider a still tank with the density linearly increasing with depth and a zero horizontal gradient at all locations, i.e. $\rho = \rho_0 + a(h - y)$, where h is the elevation of the water surface and ρ_0 is the density at the water surface. Then, in Figure 1, $\rho_C = \rho_0 + ah_1$, $\rho_D = \rho_0 + ah_2$. Integrating vertically to obtain pressure at nodes C and D the tank will be hydrostatic:

$$p = g[\rho_0(h - y) + (a/2)(h - y)^2],$$

then

$$p_C = gh_1[\rho_0 + (a/2)h_1],$$
$$p_D = gh_2[\rho_0 + (a/2)h_2].$$

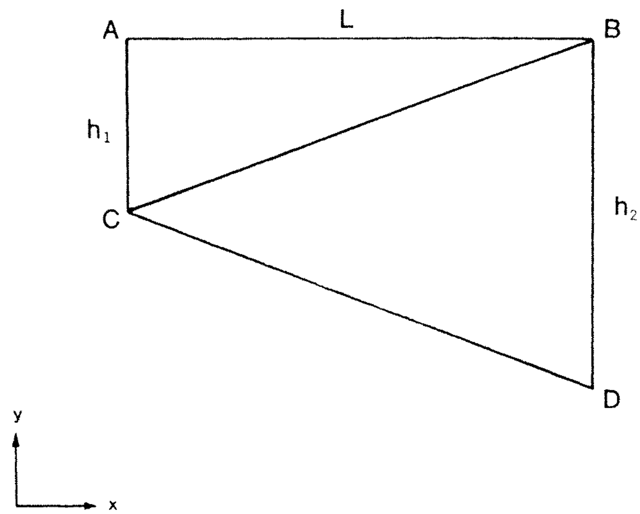

Figure 1. Typical element with non-horizontal structure

In finite element applications, where p is assumed linear, the resulting pressure gradient along the horizontal line drawn from C will be given by

$$\frac{1}{2}\left(\frac{h_1}{h_2}p_{\mathrm{D}} - p_{\mathrm{C}}\right) = \frac{gh_1}{L}\frac{a}{2}(h_2 - h_1).$$

Thus if $h_1 \neq h_2$, the pressure gradient will be non-zero and forces that create circulation will be induced.

As a demonstration of this problem, consider the network shown in Figure 2(c) which will be further defined when the example problem is discussed. It represents a reservoir at actual scale; if it is assumed that the temperature distribution is fixed and varies linearly with elevation (surface temperature of 20·0 °C and lowest temperature of 15·76 °C, with a linear equation of state relating density and temperature) then the original model induces circulation with a maximum velocity of 0·0911 fps at node 259.

These inconsistencies can be reduced with the introduction of the concept of discrepancy from hydrostatic pressure P_{d}.

Let P_{d} be defined as the discrepancy from hydrostatic pressure at an elevation y in the system which has a free surface at elevation h at this location (i.e. y and h lie on the same vertical line), i.e.

$$P_{\mathrm{D}} = p - \int_y^h \rho g \,\mathrm{d}\eta.$$

The pressure and density terms in the vertical momentum equation may be restated as

$$\begin{aligned}\frac{\partial p}{\partial y} + \rho g &= \frac{\partial P_{\mathrm{D}}}{\partial y} + \frac{\partial}{\partial y}\left(\int_y^h \rho g\,\mathrm{d}\eta\right) + \rho g,\\ &= \frac{\partial P_{\mathrm{D}}}{\partial y},\end{aligned}$$

and the pressure term of the horizontal momentum equation may be rewritten

$$\frac{\partial p}{\partial x} = \frac{\partial P_{\mathrm{D}}}{\partial x} + \frac{\partial}{\partial x}\left(\int_y^h \rho g\,\mathrm{d}\eta\right).$$

If $P_{\mathrm{H}} = \int_y^h \rho g\,\mathrm{d}\eta$, the horizontal term becomes

$$\frac{\partial p}{\partial x} = \frac{\partial P_{\mathrm{D}}}{\partial x} + \frac{\partial P_{\mathrm{H}}}{\partial x}.$$

Note that P_{H}, and thus $\partial P_{\mathrm{H}}/\partial x$, reflect the effects of complex distributions of temperature. It is also possible to use the concept of a reference density in order to reduce dependence of the magnitude of P_{H} on depth and so to eliminate the occurrence of small differences of large numbers that are encountered when computing horizontal pressure gradients. Let this reference density be ρ_{n}. Then, if

$$P_{\mathrm{H}}^* = \int_y^h g(\rho - \rho_{\mathrm{n}})\,\mathrm{d}\eta,$$

$$P_{\mathrm{H}} = g\int_y^h (\rho - \rho_n)\,\mathrm{d}\eta + \rho_{\mathrm{n}} g(h - y) = P_{\mathrm{H}}^* + \rho_{\mathrm{n}} g(h - y),$$

and

$$\frac{\partial p}{\partial x} = \frac{\partial P_{\mathrm{D}}}{\partial x} + \frac{\partial P_{\mathrm{H}}}{\partial x} = \frac{\partial P_{\mathrm{H}}^*}{\partial x} + \frac{\partial h}{\partial x}\rho_{\mathrm{n}} g + \frac{\partial P_{\mathrm{D}}}{\partial x},$$

In this form the dependent variable P_D is the discrepancy from hydrostatic pressure; it is still approximated by a linear function and mixed interpolation is thus preserved. P_H^* and $\rho_n g(h-y)$ are the components of hydrostatic pressure which may be determined directly from the density values, and may be considered almost as external tractions represented by quadratic approximations that act on the system.

For convenience of this integration the elements should be arranged so that all nodes lie on vertical lines.

If we return to our original still tank problem, with the irregular element of Figure 1, the vertical momentum equation is identically satisfied, since the pressure distribution is hydrostatic and $P_D = 0$.

In the horizontal momentum equation $\partial h/\partial x = 0$, $\partial P_D/\partial x = 0$; only $\partial P_H^*/\partial x$ can possibly exist. In the example the density is assumed to vary linearly with depth. Let $\rho_n = \rho_0$ so that $\rho = \rho_0 + a(h-y)$. Thus at C

$$P_H^* = \int_{h-h_1}^{h} g[\rho_0 + a(h-\eta) - \rho_0]\,d\eta = \frac{agh_1^2}{2}.$$

Similarly at D

$$P_H^* = \frac{agh_2^2}{2}.$$

As expected, the function P_H^* shows a simple quadratic form with respect to depth; however the form of P_H^* in this method is not limited to the linear pressure approximation used previously; in fact P_H^* can be computed using a quadratic approximation down each vertical line of the system, so that in subsequent computation for this case $\partial P_H^*/\partial x$ will be exactly zero.

In the reservoir example of Figure 2(c), previously discussed, the modified version of the program does indeed induce no circulation.

The preceding analysis has been developed for triangular elements with straight sides. For these elements the isoparametric transformation applied to linear functions generates linear functions. In the case of quadrilateral elements this relationship does not hold and the analysis above does not apply exactly. Application of these methods to the model using quadrilateral elements has shown that the amount of circulation induced can be considerably reduced, but not to zero. The evidence of this analysis suggests that triangular elements may be preferable when using this model.

Note that this modification will only function exactly even for triangles if density is described as a linear function over the element. In most applications the state equation and the temperature or salinity distributions are non-linear, so that in order to implement this scheme with a quadratic description of P_H^* some information is lost. It is believed that the use of a higher approximation of P_H^* cannot be economically justified.

THE HYDROSTATIC PRESSURE ASSUMPTION

In order to simplify the complexity of stratified flow numerous researchers have eliminated the vertical momentum equation by assuming that the vertical pressure gradient exactly matches the gravity force, 'the hydrostatic pressure assumption'. The derivation above presents a convenient vehicle for evaluation of the impact of this assumption, the magnitude of P derived in a solution is a direct measure of this approximation. It is also relatively easy to modify the computer code to incorporate the hydrostatic assumption and compare the resulting velocity regimes. The momentum equation in the vertical direction is completely eliminated and the continuity equation describing flow in terms of the two velocity components is replaced by an integrated equation that represents overall continuity in the horizontal direction. The pressure variable defined in terms of discrepancy from hydrostatic is identically zero. The only pressure parameter is the overpressure associated with the rigid lid assumption which is constant for any vertical line.

The finite element formulation can thus be written entirely in terms of horizontal velocities at points on a vertical line and a single pressure variable on each vertical line. In this process the contribution of the vertical velocities to the horizontal momentum equation is neglected. If however it is desired to compute the vertical velocities and to incorporate them into the momentum terms, it is possible to add a differentiated form of the local continuity equation to the structure. The act of differentiation allows the specification of values for vertical velocity at all boundary points in terms of horizontal velocities. Details of this approach applied to three dimensional problems have been described elsewhere by the author.[6] The equations that are solved in the hydrostatic model have the forms:

x-momentum:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]+\frac{\partial P_{\mathrm{H}}^{*}}{\partial x}+\frac{\partial h}{\partial x}\rho_{\mathrm{n}}g+\frac{\partial P_{\mathrm{D}}}{\partial x}-\varepsilon_{xx}\frac{\partial^2 u}{\partial x^2}-\varepsilon_{xy}\frac{\partial^2 u}{\partial y^2}=0,$$

overall continuity:

$$\int_{B_1}^{h}\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]\mathrm{d}\eta=0,$$

differentiated continuity

$$\frac{\partial^2 v}{\partial y^2}+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)=0.$$

In these equations P_{D}, h—the water surface elevation— and B_1—the bottom elevation—are functions of x only. In this analysis h is assumed fixed in time to form the 'rigid lid' assumption and P_{D} then represents the overpressure of the 'rigid lid' interface.

DEMONSTRATION PROBLEM

In order for a demonstration problem to be meaningful, a system with dimensions approximating those of a reservoir must be simulated. The effectiveness of the finite element model has already been demonstrated using comparisons to laboratory scale systems, and simulation of actual prototype systems such as Lake Taneycomo in Missouri. The purpose of this demonstration problem is to show that the modified system behaves appropriately on full size problems and to investigate the influence of the hydrostatic pressure assumption for reservoir simulation. Figure 2(a) presents the basic dimensions for the test case with specified upstream inflow, and withdrawal through two outlets. The reservoir has a highly variable cross-section, as is demonstrated in Figure 2(b), which shows the contours of constant width, and Figure 2(c) shows the finite element representation. In this case the rigid lid boundary condition is used to describe the water surface. The resulting water surface pressures can thus be used to evaluate the solution. The problem is based upon an actual reservoir and all flow rates, dimensions and temperature gradients are consistent with the prototype system.

For this problem the temperature distribution has been specified from measurements taken at various points in the reservoir. Temperatures vary from 17 °C at the upstream section to 16 °C at the outlets and 10·5 °C at the lowest levels. Figure 3 presents isotherms for the system. A steady state analysis is used as the basis for comparison. The flow regime is thus stratified and the velocity distribution is partly driven by the variation of density.

Any solutions to problems of this nature are strongly dependent on the eddy viscosity used. The current version of the models allows the specification of all four eddy coefficients independently for each element; however, there is seldom sufficient data to justify or predefine such a detailed description. It is commonly assumed that the magnitudes of the xy and yy terms are functions of density gradient and velocity; see for example Reference 3. For this application it has been assumed that the nominal values of these coefficients are unifom throughout the reservoir, but that the actual values of E_{xy} and E_{yy} are scaled to a function of the stability S,

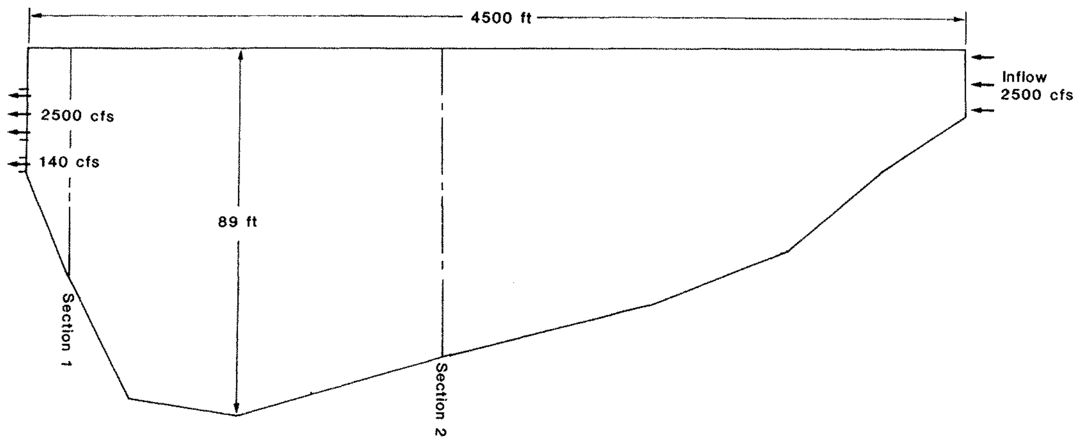

Figure 2(a). Dimension demonstration problem

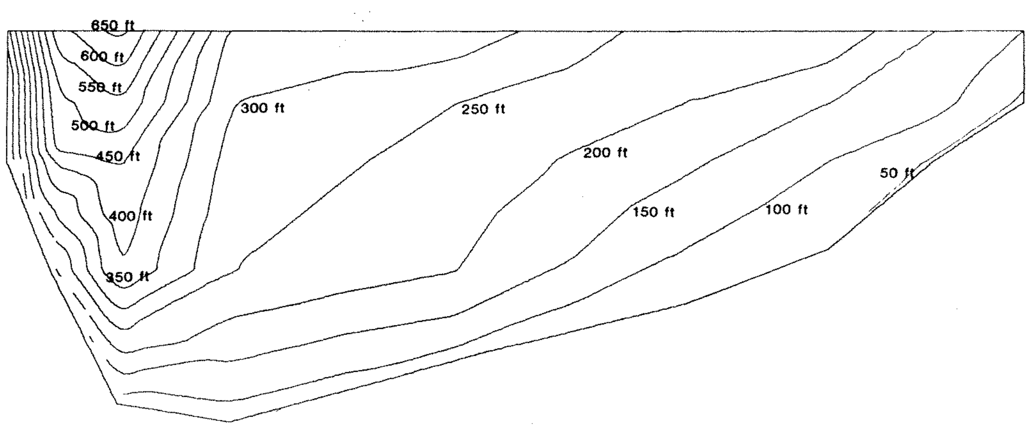

Figure 2(b). Contours of constant width—demonstration problem

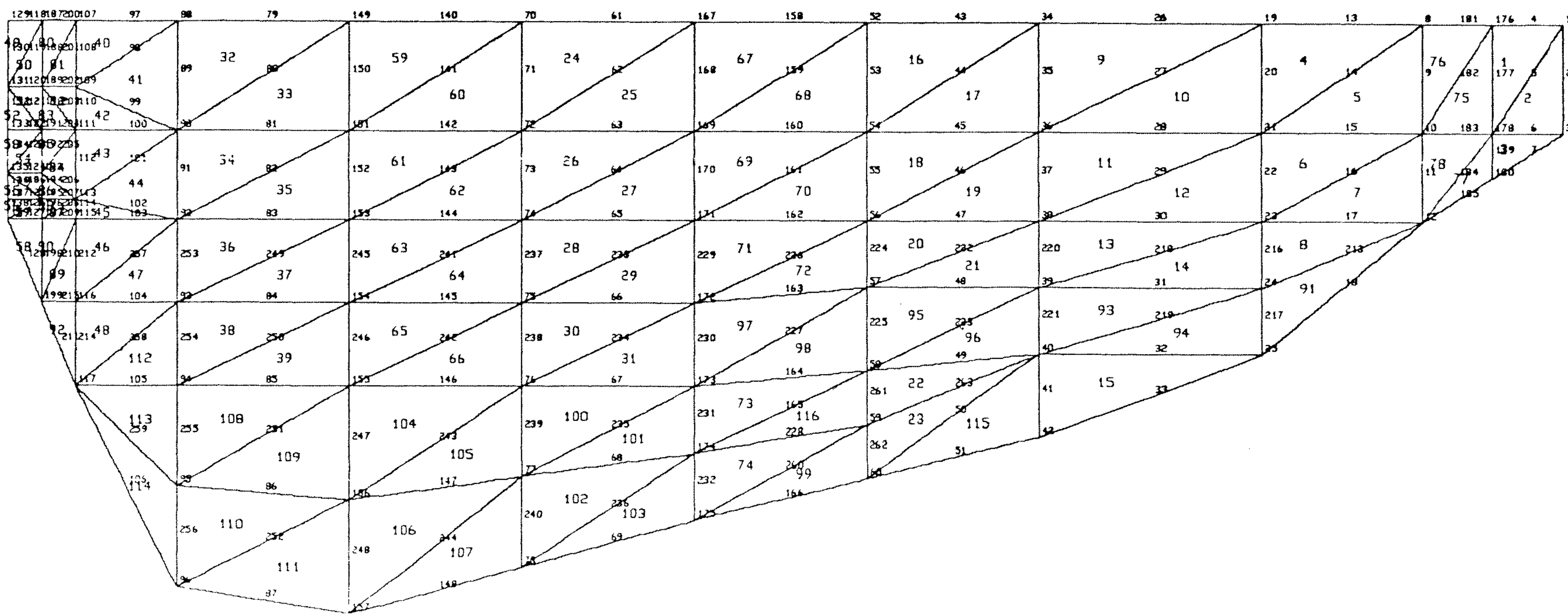

Figure 2(c). Finite element network–demonstration problem

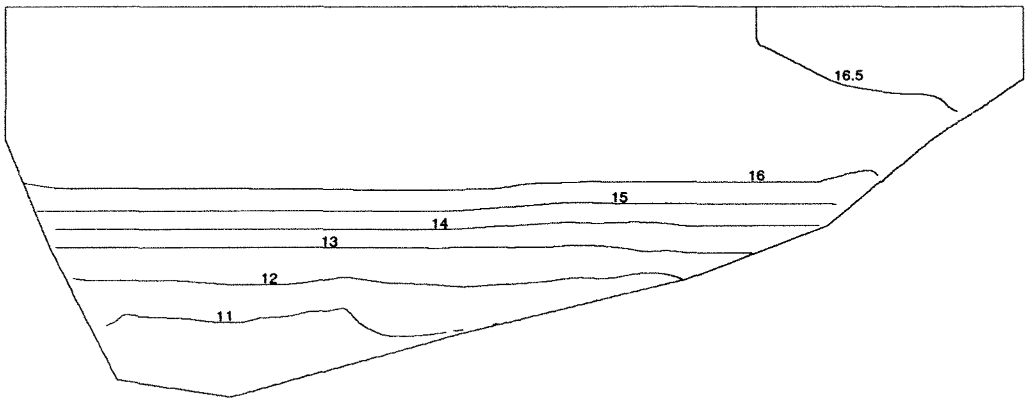

Figure 3. Contours of constant temperature—demonstration problem

where $S = -(1/\rho)(\partial\rho/\partial y)$. The exact form of the dependence is taken from analysis of reservoir data undertaken by Water Resources Engineers[7] while developing a one dimensional model for predicting temperatures in deep reservoirs. Specifically

$$\varepsilon_{xy} = \varepsilon_{xyn} \qquad \text{for } S < 3{\cdot}048 \times 10^{-6}$$

and

$$\varepsilon_{xy} = \varepsilon_{xyn} \times 1{\cdot}3766 \times 10^{-7} S^{-0{\cdot}7} \quad \text{for } S > 3{\cdot}048 \times 10^{-6}.$$

The nominal values used were

$$\varepsilon_{xxn} = \varepsilon_{yxn} = 100\,\text{lb-s/ft}^2$$
$$\varepsilon_{xyn} = \varepsilon_{yyn} = 0{\cdot}25\,\text{lb-s/ft}^2$$

A linear equation of state was used for this analysis, taking the form $\rho = 1{\cdot}938641 - 3{\cdot}055722 \times 10^{-4}(\mathrm{T} - 13{\cdot}625)$. In order to evaluate the model three different cases have been evaluated. In case 1 the flow regime is assumed to be fully homogeneous, that is the influences of temperature stratification are ignored. In case 2 the stratified flow regime was simulated with full equations and, finally, in case 3 the stratified flow regime was modelled using the hydrostatic approximation. Case 1 was included in this analysis to establish a baseline flow regime that

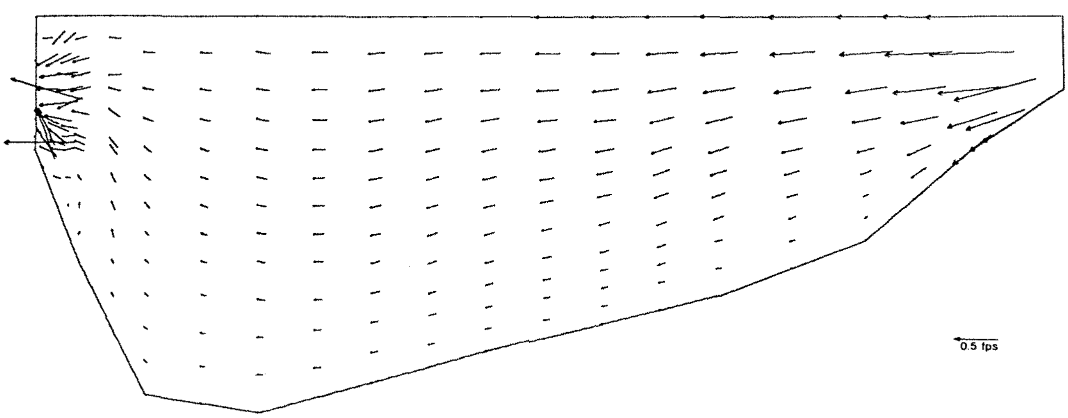

Figure 4. Reservoir velocities—homogeneous case

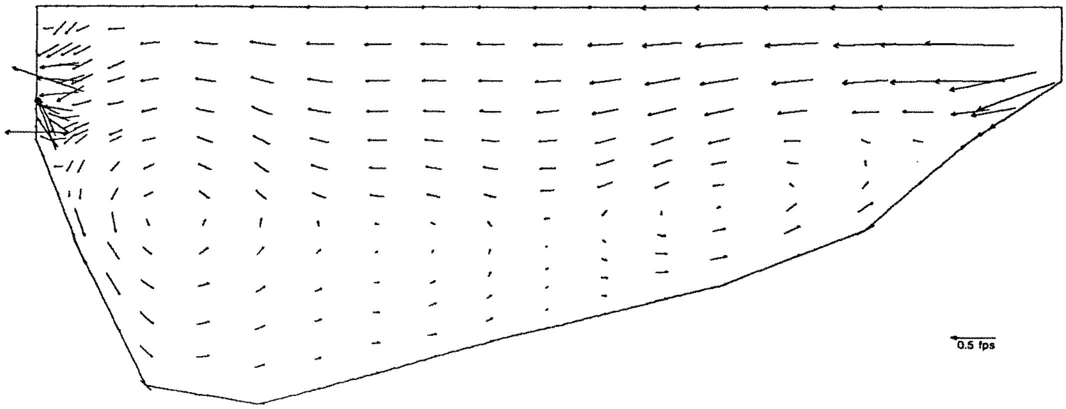

Figure 5. Reservoir velocities—stratified case

shows by comparison the influence of temperature on the flow regimes for the other two cases. The overall flow structure is presented as three sets of velocity vectors in Figures 4, 5, and 6. A comparison of these Figures shows that overall distribution is as expected, with the horizontal flow case showing no reversal of flow and the two stratified cases showing strong reversals below the level of the thermocline. The solutions for the hydrostatic and full momentum equations are generally very similar; the only differences occur in the vicinity of the withdrawal structure. It is important to point out that these Figures are to a distorted vertical scale, so that the magnitude of the very small vertical velocity components is highly exaggerated. In order to make further comparison of the flow regimes, Table I presents the velocities for all three cases on a vertical section close to the dam and at a point further upstream; exact locations are shown in Figure 2(a). Note the similarity of horizontal velocity distributions, particularly for the more upstream location; however, magnitudes of the reverse flow are slightly larger for the hydrostatic equation solution, probably reflecting the lack of energy loss in the vertical momentum equation. The vertical velocities show some deviations particularly close to the dam, but the magnitudes are very small. When the stratified flow velocities are compared with the homogeneous flow velocities it is clear that the hydrostatic assumption adequately captures the essence of the stratified flow regime.

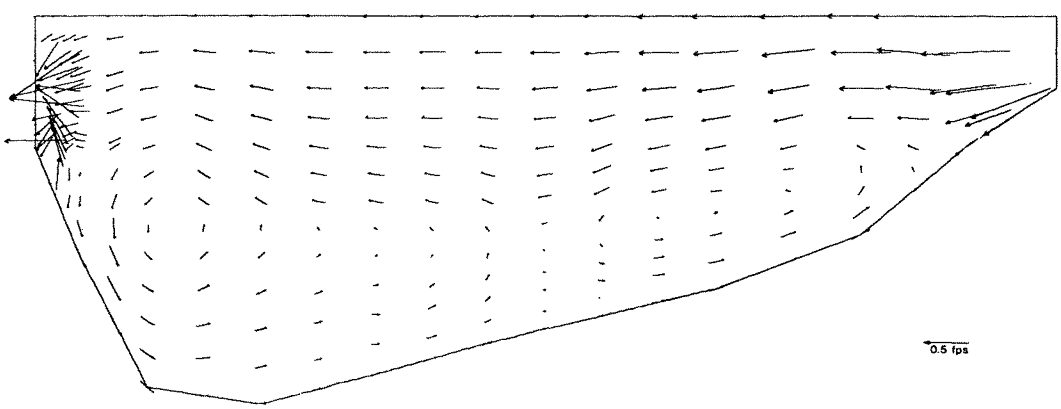

Figure 6. Reservoir velocities—stratified case: hydrostatic assumption

Table I. Demonstration case: velocities on vertical sections

Node	Horizontal velocities (fps) Homogeneous flow	Full equation	Hydrostatic approximation	Vertical velocities (fps) Homogeneous flow	Full equation	Hydrostatic approximation
Section 1						
187	−0·10160	−0·09860	−0·12260	0·00000	0·00000	0·00000
188	−0·13720	−0·13420	−0·15610	−0·00778	−0·00770	−0·00330
189	−0·28480	−0·28210	−0·29790	−0·00776	−0·00806	−0·01076
190	−0·42840	−0·42690	−0·44320	−0·00307	−0·00310	−0·00770
191	−0·50030	−0·49990	−0·49630	0·00116	0·00134	0·00329
192	−0·43410	−0·43450	−0·47240	−0·00195	−0·00222	0·01727
193	−0·30810	−0·31040	−0·32950	0·00346	0·00276	0·02341
194	−0·23030	−0·23340	−0·23720	0·00520	0·00451	0·02202
195	−0·20120	−0·20490	−0·19100	0·00786	0·00704	0·02120
196	−0·18110	−0·18500	−0·16090	0·00301	0·00172	0·02204
197	−0·16010	−0·16440	−0·11880	0·00095	−0·00069	0·02237
198	−0·08570	−0·09280	0·02270	0·00057	−0·00050	0·01880
199	−0·00120	−0·01320	−0·08020	0·00015	0·00164	0·01002
Section 2						
167	−0·23160	−0·29800	−0·29780	0·00000	0·00000	0·00000
168	−0·22570	−0·28610	−0·28600	−0·00053	0·00036	0·00043
169	−0·20940	−0·25500	−0·25490	−0·00112	0·00039	0·00102
170	−0·19060	−0·22250	−0·22250	−0·00162	0·00071	0·00136
171	−0·16860	−0·18750	−0·18740	−0·00183	0·00100	0·00180
229	−0·14720	−0·15680	−0·15620	−0·00198	0·00147	0·00228
172	−0·12650	−0·12950	−0·12800	−0·00183	0·00157	0·00281
230	−0·10780	−0·04050	−0·03850	−0·00165	0·00255	0·00320
173	−0·09220	0·00940	0·01230	−0·00143	0·00230	0·00319
231	−0·08250	0·03390	0·03660	−0·00128	0·00260	0·00307
174	−0·07540	0·05170	0·05480	−0·00119	0·00205	0·00277
232	−0·07100	0·06570	0·06910	−0·00109	0·00178	0·00207
175	−0·06940	0·07580	0·07990	−0·00092	0·00101	0·00106

Table II. Surface pressures by node

Node	Stratified flow Homogeneous flow, Slugs/ft^2	Full solution, Slugs/ft^2	Hydrostatic case, Slugs/ft^2
1	−4·196	−3·910	−3·882
176	−1·098	−0·802	−0·748
8	−0·372	−0·145	−0·096
19	−0·008	0·078	0·123
34	0·059	0·107	0·158
52	0·089	0·126	0·170
167	0·109	0·162	0·208
70	0·119	0·125	0·167
149	0·115	0·079	0·125
88	0·127	0·071	0·117
107	0·128	0·099	0·150
187	0·234	0·228	0·254
129	0·000	0·000	0·000

The velocity regimes and the pressure distributions along the water surface show some differences for the two cases. Table II presents a comparison of values for each case. However, the magnitude of surface elevation change for these cases is less than 1/100 ft. It appears that the primary driving force that generates circulation is a combination of momentum and continuity, the proportions of which vary for the three cases, and that from a measurement perspective these surface slope values are indistinguishable.

CONCLUDING REMARKS

The purpose of this paper has been to review recent experience with laterally integrated models and to indicate some of the steps necessary to ensure satisfactory behaviour. As the demonstration problem shows, the modified model gives satisfactory results and the generally accepted hydrostatic assumption appears reasonable for reservoir scale applications, although there are some differences in magnitude of velocities, except in the vicinity of the outlet works, where the flow is driven by larger pressure gradients.

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